Research Methods

Factor Analysis with R

Andy Gillett

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Factor Analysis

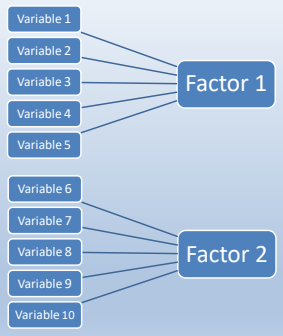
Factor analysis is a statistical technique for analysing the correlations between a number of variables in order to reduce them to a smaller number of underlying dimensions, called factors, and to determine the correlation of each of the original variables with each factor.

2

Factor Analysis

3

Factor Analysis



The amount of the total variance explained by that factor is called the **eigenvalue** of a factor.

The extent to which a particular variable contributes to a given factor is the **factor loading** of a variable.

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Factor Analysis

Job Satisfaction

interest	Job Interest
supervis	Relationship with Supervisor
facilit	Facilities in the Workplace
peerrel	Relationship with Peers/Colleagues
persdev	Opportunities for Personal Development & Promotion
workhours	Hours of Work
holidays	Holidays

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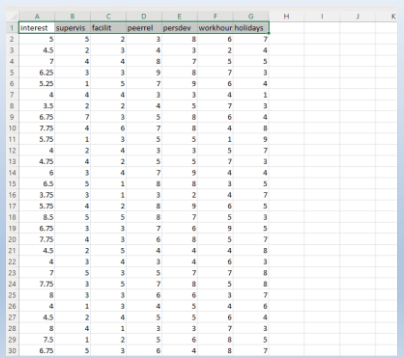
Factor Analysis



interest	supervis	facilit	peerrel	persdev	workhour	holidays
5	5	2	3	8	6	7
4.5	2	3	4	3	2	4
7	4	4	8	7	5	5
6.25	3	3	9	8	7	3
5.25	1	5	7	9	6	4
4	4	4	3	3	4	1
3.5	2	2	4	5	7	3
6.75	7	3	5	8	6	4
7.75	4	6	7	8	4	8
5.75	1	3	5	5	1	9
4	2	4	3	3	5	7
4.75	4	2	5	5	7	3
6	3	4	7	9	4	4
6.5	5	1	8	8	3	5
3.75	3	1	3	2	4	7
5.75	4	2	8	9	6	5
8.5	5	5	8	7	5	3
6.75	3	3	7	6	9	5
7.75	4	3	6	8	5	7
4.5	2	5	4	4	4	8
4	3	4	3	4	6	3
7	5	3	5	7	7	8
7.75	3	5	7	8	5	8
8	3	3	6	6	3	7
4	1	3	4	5	4	6
4.5	2	4	5	5	6	4
8	4	1	3	3	7	3
7.5	1	2	5	6	8	5
6.75	5	3	6	4	8	7

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Factor Analysis

job-satisfaction.csv

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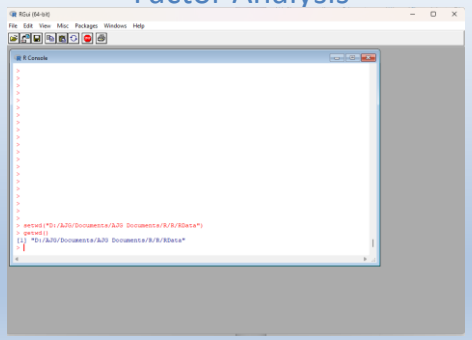
Factor Analysis



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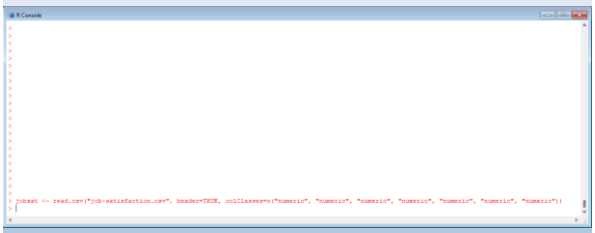
8

Factor Analysis



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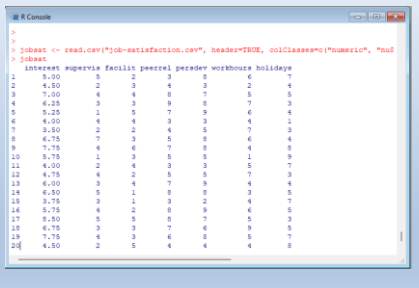

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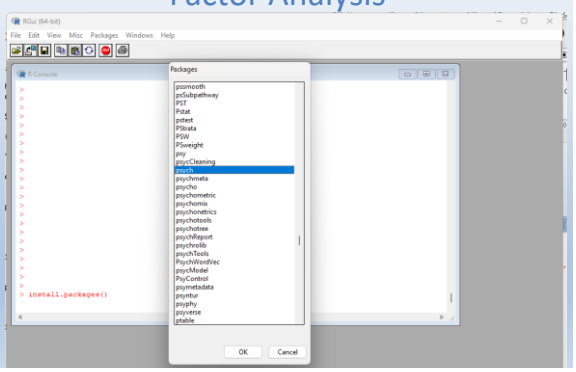
Factor Analysis



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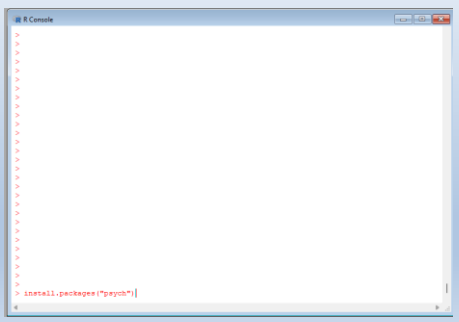
Factor Analysis



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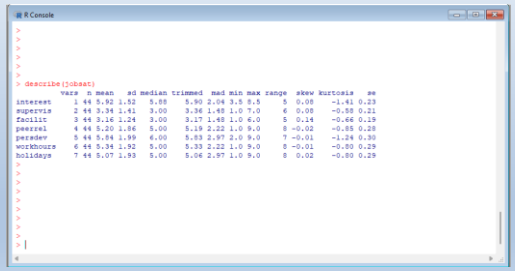
12

Factor Analysis



```
> describe(jobsat)
          vars  n mean   sd median trimmed  mad min max range  skew kurtosis   se
interest     1 44 5.92 1.52   5.88    5.90 2.04 3.5 8.5     5  0.08    -1.41 0.23
supervis     2 44 3.34 1.41   3.00    3.36 1.48 1.0 7.0     6  0.08    -0.58 0.21
facilit      3 44 3.16 1.24   3.00    3.17 1.48 1.0 6.0     5  0.14    -0.66 0.19
peerrel      4 44 5.20 1.86   5.00    5.19 2.22 1.0 9.0     8 -0.02    -0.85 0.28
persdev      5 44 5.84 1.99   6.00    5.83 2.97 2.0 9.0     7 -0.01    -1.24 0.30
workhours    6 44 5.34 1.92   5.00    5.33 2.22 1.0 9.0     8 -0.01    -0.80 0.29
holidays     7 44 5.07 1.93   5.00    5.06 2.97 1.0 9.0     8  0.02    -0.80 0.29
```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Factor Analysis

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          vars  n mean   sd median trimmed  mad min max range  skew kurtosis   se
interest     1 44 5.92 1.52   5.88    5.90 2.04 3.5 8.5     5  0.08    -1.41 0.23
supervis     2 44 3.34 1.41   3.00    3.36 1.48 1.0 7.0     6  0.08    -0.58 0.21
facilit      3 44 3.16 1.24   3.00    3.17 1.48 1.0 6.0     5  0.14    -0.66 0.19
peerrel      4 44 5.20 1.86   5.00    5.19 2.22 1.0 9.0     8 -0.02    -0.85 0.28
persdev      5 44 5.84 1.99   6.00    5.83 2.97 2.0 9.0     7 -0.01    -1.24 0.30
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holidays     7 44 5.07 1.93   5.00    5.06 2.97 1.0 9.0     8  0.02    -0.80 0.29
>
```


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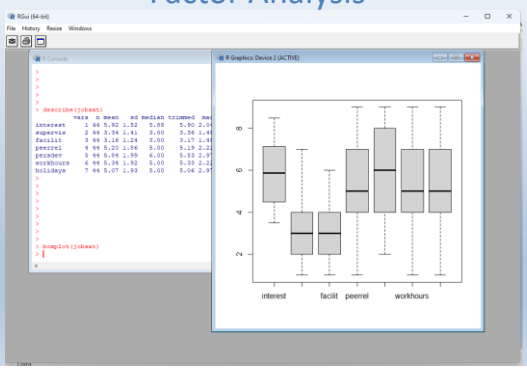
Factor Analysis



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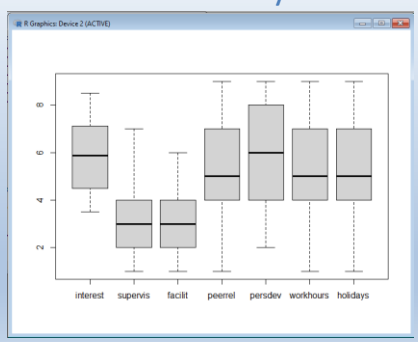
Factor Analysis



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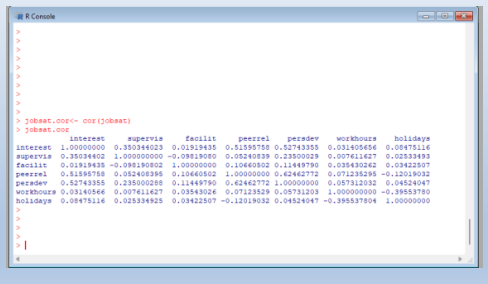
Factor Analysis



```
> jobsat.cor<- cor(jobsat)
> jobsat.cor
             interest     supervis     facilit     peerrel     persdev    workhours    holidays
interest   1.00000000  0.350344023  0.01919435  0.51595758  0.52743355  0.031405656  0.08475116
supervis   0.35034402  1.000000000 -0.09819080  0.05240839  0.23500029  0.007611627  0.02533493
facilit    0.01919435 -0.098190802  1.00000000  0.10660502  0.11449790  0.035430262  0.03422507
peerrel    0.51595758  0.052408395  0.10660502  1.00000000  0.62462772  0.071235295 -0.12019032
persdev    0.52743355  0.235000288  0.11449790  0.62462772  1.00000000  0.057312032  0.04524047
workhours  0.03140566  0.007611627  0.03543026  0.07123529  0.05731203  1.000000000 -0.39553780
holidays   0.08475116  0.025334925  0.03422507 -0.12019032  0.04524047 -0.395537804  1.00000000
>
```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Factor Analysis

```
> jobsat.cor<- cor(jobsat)
> jobsat.cor
             interest     supervis     facilit     peerrel     persdev    workhours    holidays
interest   1.00000000  0.350344023  0.01919435  0.51595758  0.52743355  0.031405656  0.08475116
supervis   0.35034402  1.000000000 -0.09819080  0.05240839  0.23500029  0.007611627  0.02533493
facilit    0.01919435 -0.098190802  1.00000000  0.10660502  0.11449790  0.035430262  0.03422507
peerrel    0.51595758  0.052408395  0.10660502  1.00000000  0.62462772  0.071235295 -0.12019032
persdev    0.52743355  0.235000288  0.11449790  0.62462772  1.00000000  0.057312032  0.04524047
workhours  0.03140566  0.007611627  0.03543026  0.07123529  0.05731203  1.000000000 -0.39553780
holidays   0.08475116  0.025334925  0.03422507 -0.12019032  0.04524047 -0.395537804  1.00000000
```


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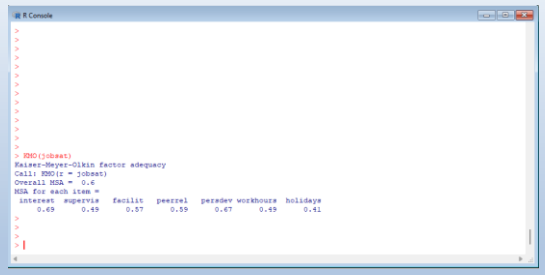
Factor Analysis



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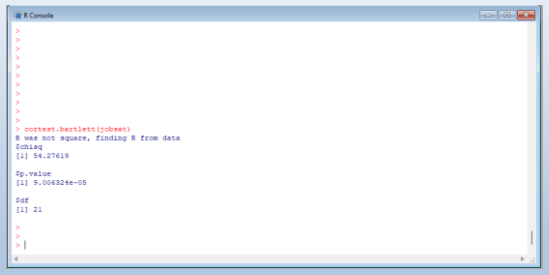
Factor Analysis



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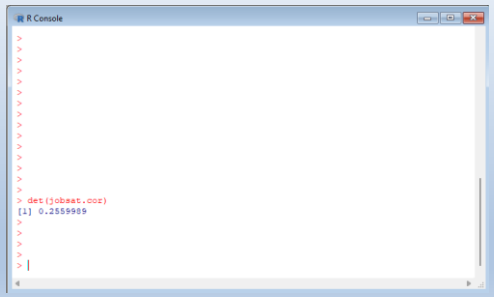
Factor Analysis



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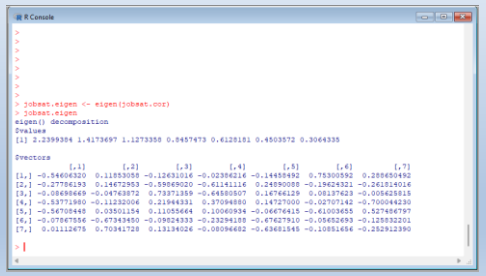
Factor Analysis



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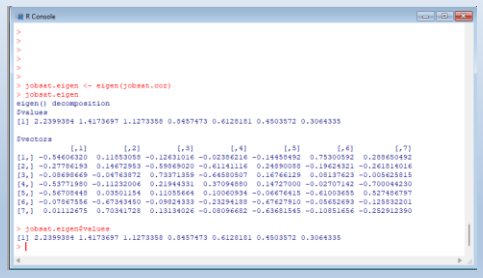
Factor Analysis



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Factor Analysis

```
> jobsat.eigen$values
[1] 2.2399384 1.4173697 1.1273358 0.8457473 0.6128181 0.4503572 0.3064335
>
```

Tabachnick and Fidell (2013) recommend that researchers adopt an exploratory approach, experimenting with different numbers of factors until a satisfactory solution is found.

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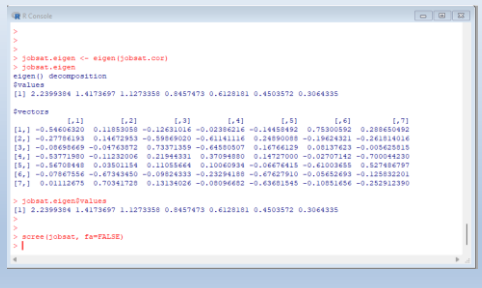
Factor Analysis



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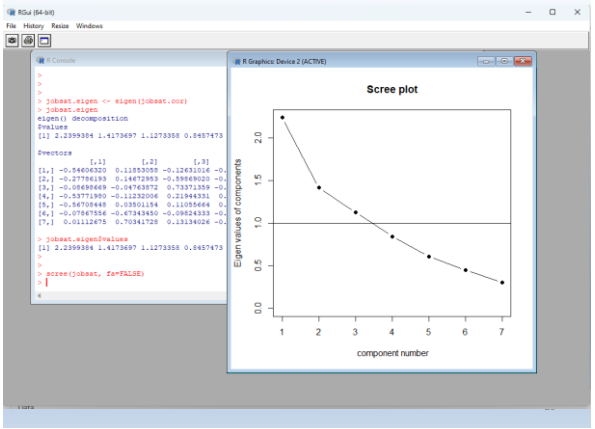

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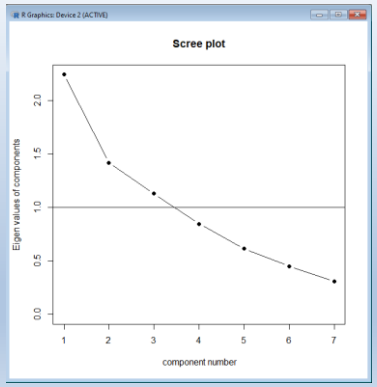
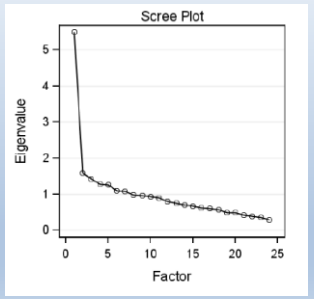


27

Factor Analysis



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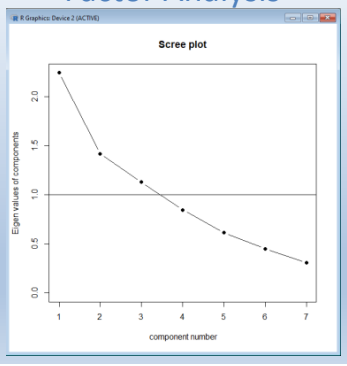
Factor Analysis



29

Factor Analysis

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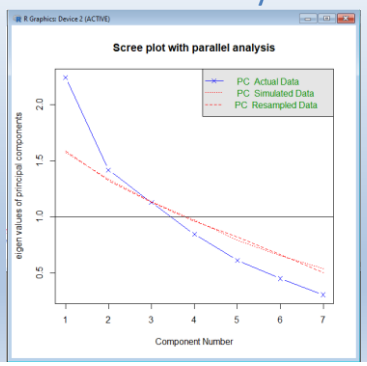
Factor Analysis



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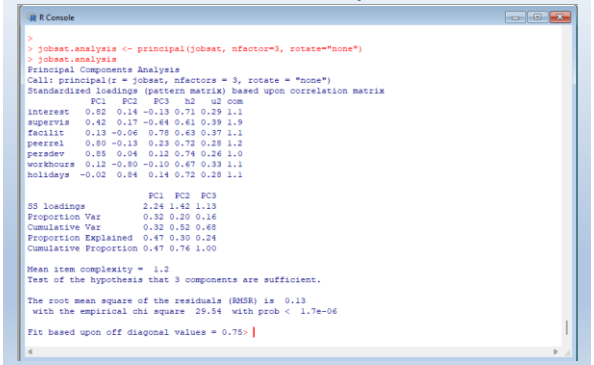
Factor Analysis



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Factor Analysis

```
> jobsat.analysis
Principal Components Analysis
Call: principal(r = jobsat, nfactors = 3, rotate = "none")
Standardized loadings (pattern matrix) based upon correlation matrix
             PC1   PC2   PC3   h2   u2 com
interest    0.82  0.14 -0.13 0.71 0.29 1.1
supervis    0.42  0.17 -0.64 0.61 0.39 1.9
facilit     0.13 -0.06  0.78 0.63 0.37 1.1
peerrel     0.80 -0.13  0.23 0.72 0.28 1.2
persdev     0.85  0.04  0.12 0.74 0.26 1.0
workhours   0.12 -0.80 -0.10 0.67 0.33 1.1
holidays   -0.02  0.84  0.14 0.72 0.28 1.1

                       PC1  PC2  PC3
SS loadings           2.24 1.42 1.13
Proportion Var        0.32 0.20 0.16
Cumulative Var        0.32 0.52 0.68
Proportion Explained  0.47 0.30 0.24
Cumulative Proportion 0.47 0.76 1.00

Mean item complexity =  1.2
Test of the hypothesis that 3 components are sufficient.

The root mean square of the residuals (RMSR) is  0.13
 with the empirical chi square  29.54  with prob <  1.7e-06

Fit based upon off diagonal values = 0.75>
```


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Factor Analysis

	PC1	PC2	PC3	h2	u2	com
interest	0.82	0.14	-0.13	0.71	0.29	1.1
supervis	0.42	0.17	-0.64	0.61	0.39	1.9
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persdev	0.85	0.04	0.12	0.74	0.26	1.0
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	PC1	PC2	PC3
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35

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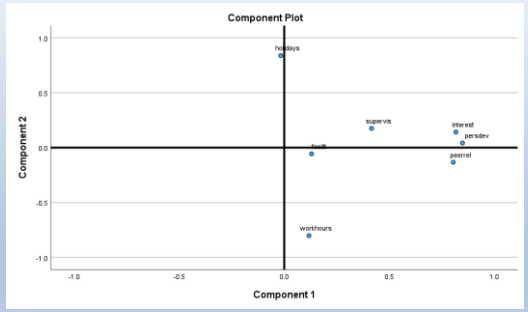
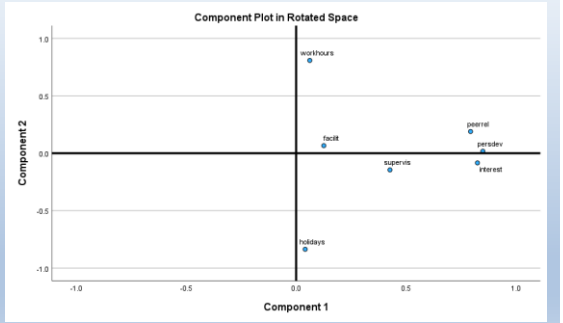


36

Factor Analysis



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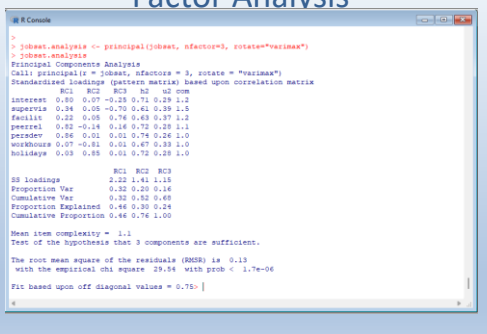
Factor Analysis



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Factor Analysis

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> jobsat.analysis
Principal Components Analysis
Call: principal(r = jobsat, nfactors = 3, rotate = "varimax")
Standardized loadings (pattern matrix) based upon correlation matrix
              RC1   RC2   RC3   h2   u2 com
interest     0.80  0.07 -0.25 0.71 0.29 1.2
supervis     0.34  0.05 -0.70 0.61 0.39 1.5
facilit      0.22  0.05  0.76 0.63 0.37 1.2
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Proportion Var	0.32	0.20	0.16
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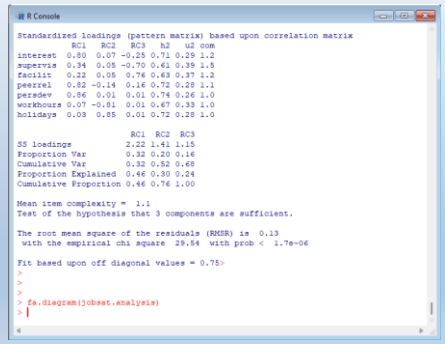

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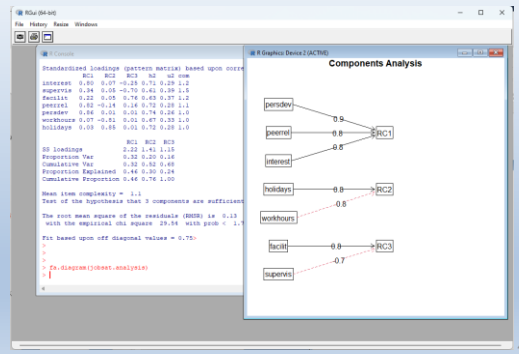
Factor Analysis



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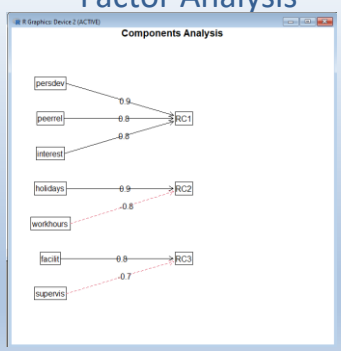
Factor Analysis



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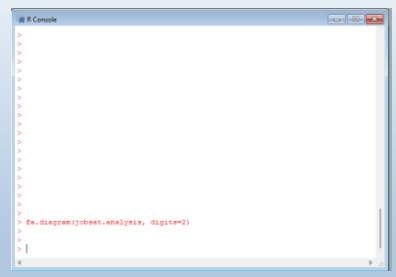
Factor Analysis



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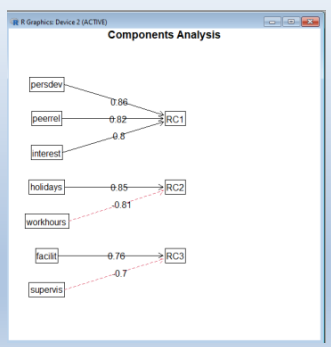
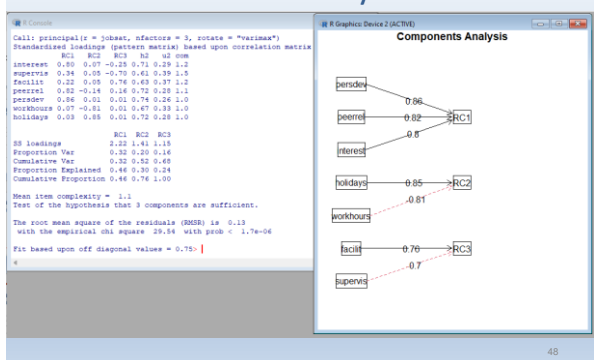
Factor Analysis



47

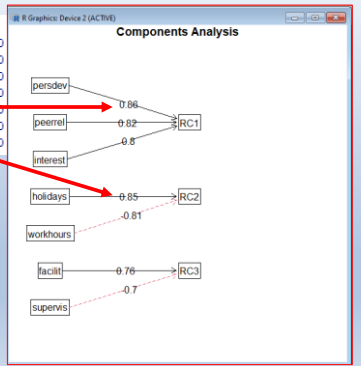
Factor Analysis



48

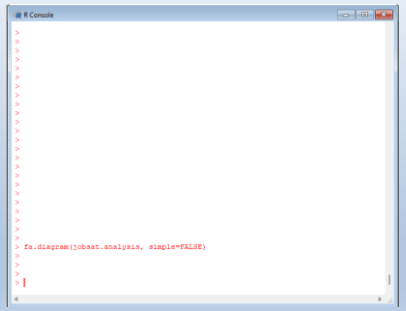
Factor Analysis

```
            RC1   RC2   RC3
interest   0.80  0.07 -0.25 0
supervis   0.34  0.05 -0.70 0
facilit    0.22  0.05  0.76 0
peerrel    0.82 -0.14  0.16 0
persdev    0.86 -0.01  0.01 0
workhours  0.07 -0.81  0.01 0
holidays   0.03  0.85 -0.01 0
```

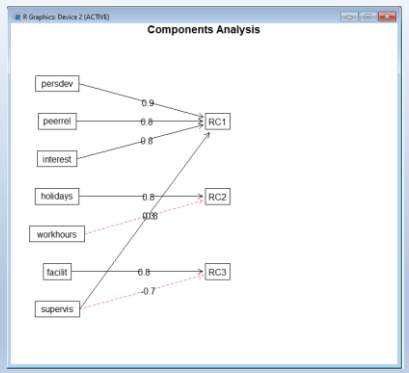


49

Factor Analysis

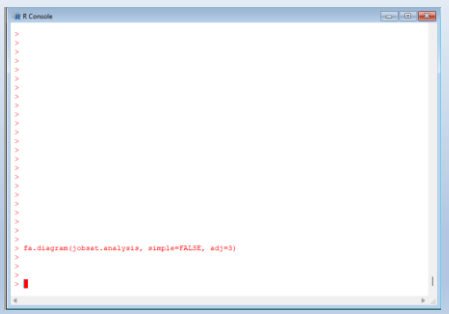


50



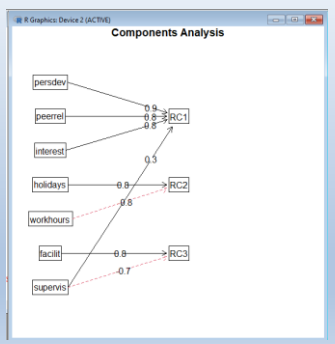
51

Factor Analysis



52

Factor Analysis



53

Factor Analysis

fa.diagram(jobsat.analysis, simple=FALSE)

fa.diagram(jobsat.analysis, digits=3)
fa.diagram(jobsat.analysis, cut=.7)

fa.diagram(jobsat.analysis, simple=FALSE, l.cex=3)
fa.diagram(jobsat.analysis, simple=FALSE, adj=4)

54

Factor Analysis

```
> jobsat.cor<- cor(jobsat)
> jobsat.cor
             interest    supervis     facilit    peerrel    persdev    workhours    holidays
interest   1.00000000  0.35034402  0.01919435  0.51595758  0.52743355  0.03140566  0.08475116
supervis   0.35034402  1.00000000 -0.09819080  0.05240839  0.23500029  0.00761127  0.02533493
facilit    0.01919435 -0.09819080  1.00000000  0.10660502  0.11449790  0.03543026  0.03422507
peerrel    0.51595758  0.05240839  0.10660502  1.00000000  0.62462772  0.07123529 -0.12019032
persdev    0.52743355  0.23500029  0.11449790  0.62462772  1.00000000  0.05731203  0.04524047
workhours  0.03140566  0.00761127  0.03543026  0.07123529  0.05731203  1.00000000 -0.39553780
holidays   0.08475116  0.02533493  0.03422507 -0.12019032  0.04524047 -0.39553780  1.00000000
```


55

Factor Analysis

```
> factor.model(jobsat.analysis$loadings)
             interest    supervis     facilit    peerrel    persdev
interest   0.70581523  0.44976693 -0.006082189  0.60759352 0.68376763
supervis   0.44976693  0.60752577 -0.450969052  0.16320641 0.28561368
facilit   -0.006082189 -0.45096905  0.427050601  0.29386666 0.19957544
peerrel    0.60759352  0.16320641  0.293866655  0.71983011 0.70480655
persdev    0.68376763  0.28561368  0.199575437  0.70480655 0.73584677
workhours -0.002916992 -0.02478013 -0.020460191  0.17766759 0.05427318
holidays   0.08586354  0.05071966  0.058973085 -0.09289329 0.03714254
             workhours    holidays
interest  -0.002916992  0.08586354
supervis  -0.024780128  0.05071966
facilit   -0.020460191  0.05897308
peerrel    0.177667586 -0.09289329
persdev    0.054273184  0.03714254
workhours  0.667542684 -0.68792299
holidays  -0.687922987  0.72103284
```

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Factor Analysis

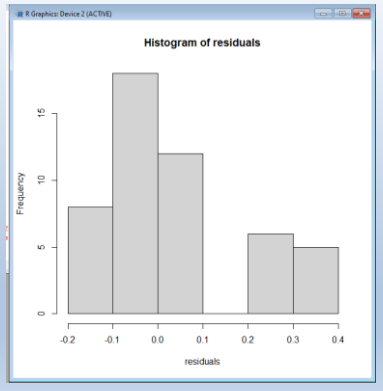
```
> factor.residuals(jobsat.cor, jobsat.analysis$loadings)
               interest     supervis      facilit      peerrel      persdev     workhours
interest   0.294184769 -0.09942291  0.02527654 -0.09163594 -0.156334076  0.034322648
supervis  -0.099422908  0.39247423  0.35277825 -0.11079802 -0.050613391  0.032391756
facilit    0.025276543  0.35277825  0.37294940 -0.18726164 -0.085077542  0.055890454
peerrel   -0.091635944 -0.11079802 -0.18726164  0.28016989 -0.080178826 -0.106432292
persdev   -0.156334076 -0.05061339 -0.08507754 -0.08017883  0.264153228  0.009038848
workhours  0.034322648  0.03239176  0.05589045 -0.10643229  0.009038848  0.332457316
holidays  -0.001112388 -0.02538474 -0.02474801 -0.02729702  0.008097932  0.292385183
              holidays
interest  -0.001112388
supervis  -0.025384735
facilit   -0.024748013
peerrel   -0.027297024
persdev    0.008097932
workhours  0.292385183
holidays   0.278967164
```


57

Factor Analysis

```
> jobsat.residuals <- factor.residuals(jobsat.cor, jobsat.analysis$loadings)
>
> hist(jobsat.residuals)
```


58



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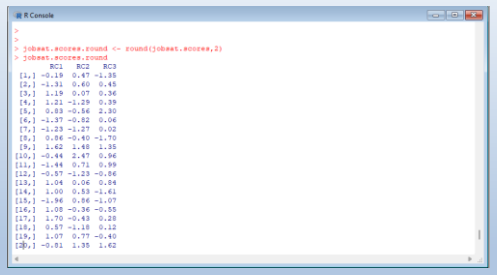
Factor Analysis

```
> jobsat.analysis <- principal(jobsat, nfactor=3, rotate="varimax", scores=TRUE)
> jobsat.scores <- jobsat.analysis$scores
> jobsat.scores
              RC1         RC2         RC3
 [1,] -0.18854038  0.46799570 -1.34793309
 [2,] -1.31070359  0.60149261  0.44615175
 [3,]  1.18992990  0.06525948  0.35775220
 [4,]  1.20526080 -1.28799222  0.39434513
 [5,]  0.82677855 -0.55900842  2.30313936
 [6,] -1.37332130 -0.82065997  0.05629439
 [7,] -1.22925508 -1.27489665  0.02452626
 [8,]  0.85647530 -0.39922297 -1.69859158
 [9,]  1.62011669  1.48065201  1.35463729
[10,] -0.44318646  2.46614439  0.96045788
[11,] -1.43861786  0.70777653  0.98937222
[12,] -0.56768414 -1.23086204 -0.86320340
[13,]  1.03758599  0.06256920  0.84070601
[14,]  0.99712365  0.53383127 -1.60999067
[15,] -1.95600516  0.86307386 -1.06695554
[16,]  1.07739407 -0.35669931 -0.34541646
[17,]  1.70230123 -0.43090617  0.28160223
[18,]  0.57429548 -1.17961296  0.11542461
```

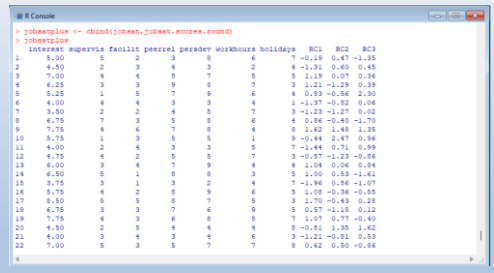

60

Factor Analysis



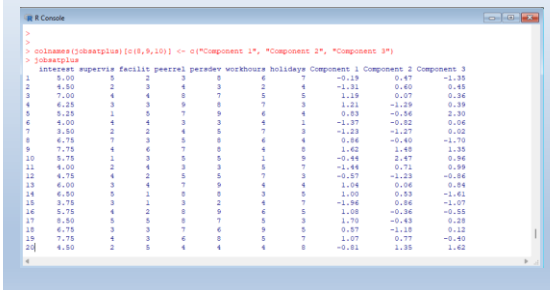
61

Factor Analysis



62

Factor Analysis



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Factor Analysis

Factor analysis, using principal components extraction and varimax rotation, was performed on the data. Three factors were extracted, Factor 1 explaining 32% of the variance in the original measures; Factor 2 explaining 20% and Factor 3 explaining an additional 16% cent of the variance. The job interest, relationship with colleagues and opportunities for personal development variables were found to have high factor loadings (.80, .82, .86) on Factor 1, hours of work and holidays had high factor loadings (.81, .85) on Factor 2 and facilities and relationship with supervisor had quite high loadings on Factor 3 (.70 - .76).

Based on Colman & Pulford (2008).

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Factor Analysis

Factor Analysis with R - 2

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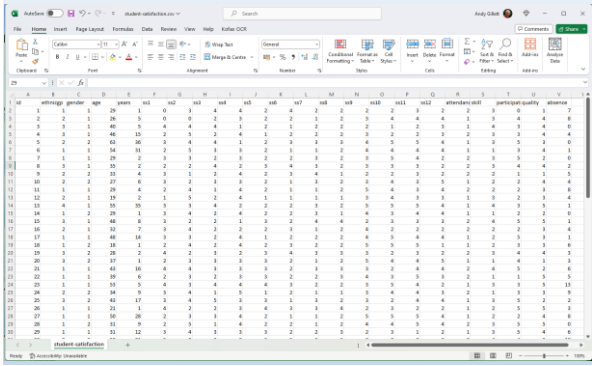
Factor Analysis

Student Satisfaction

id	Identity
ethnicgp	Ethnic Group
age	Age
gender	Gender
years	Number of Years in Institution
ss1-ss12	Twelve Measures of Satisfaction of Course
attendance	Number of Hours Attending
skill	Initial Skill Level
participation	Class Participation
quality	Quality of Work
absence	Number of Absences

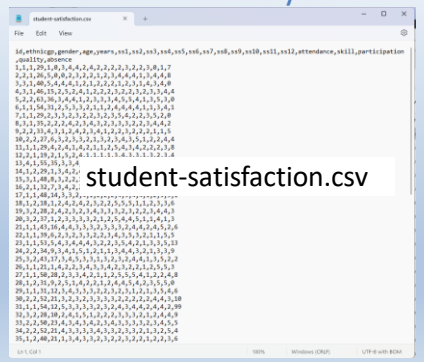
66

Factor Analysis



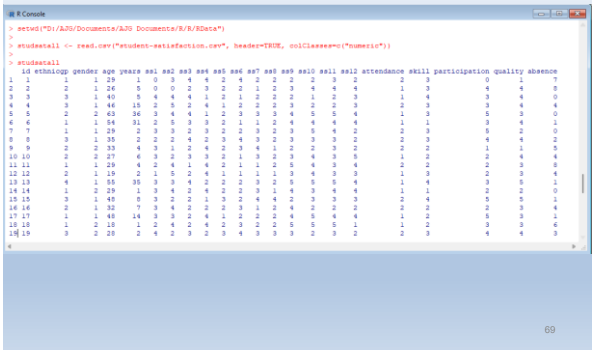
67

Factor Analysis



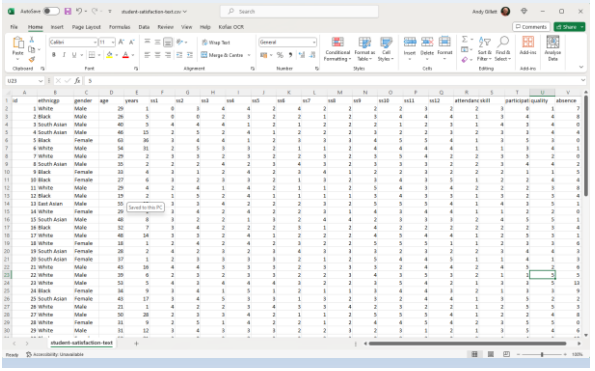
68

Factor Analysis



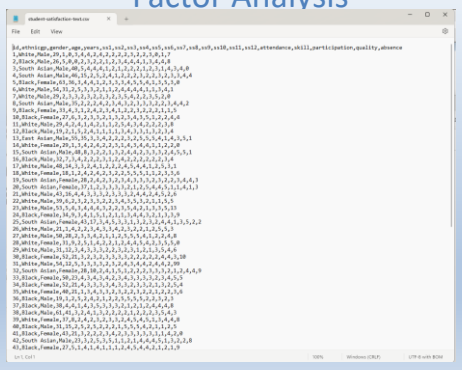
69

Factor Analysis



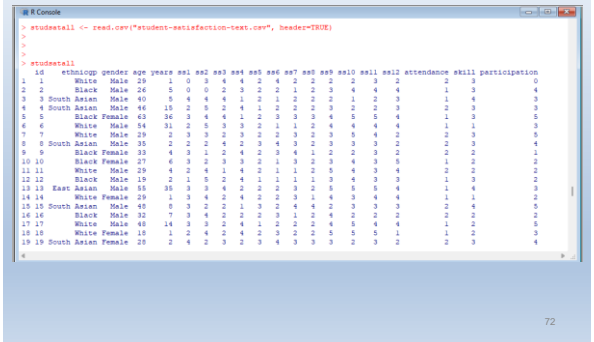
70

Factor Analysis



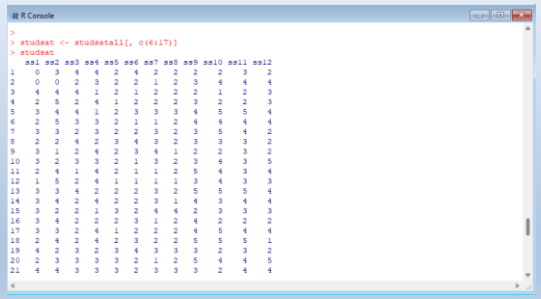
71

Factor Analysis



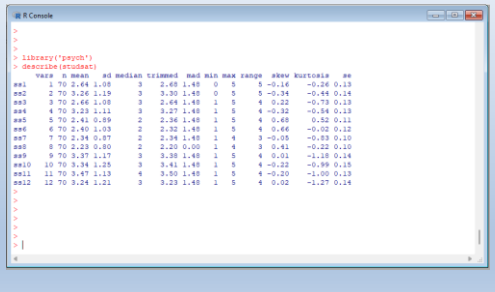
72

Factor Analysis



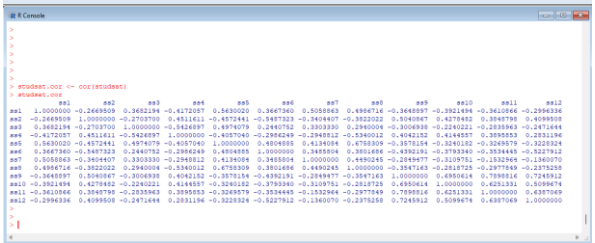
73

Factor Analysis



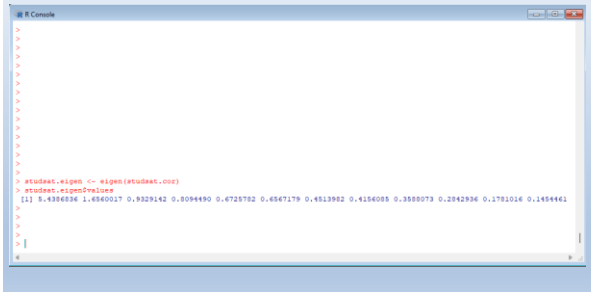
74

Factor Analysis



75

Factor Analysis



76

Factor Analysis

```
studsat.eigen$values
[1] 5.4386836 1.6560017 0.9329142 0.8094490 0.6725782 0.6567179 0.4513982
[8] 0.4156085 0.3588073 0.2842936 0.1781016 0.1454461
```

Tabachnick and Fidell (2013) recommend that researchers adopt an exploratory approach, experimenting with different numbers of factors until a satisfactory solution is found.

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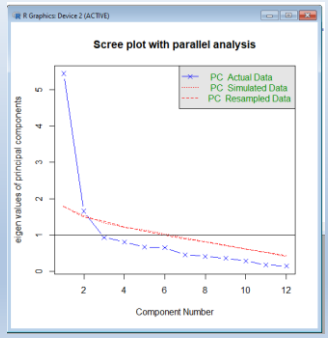
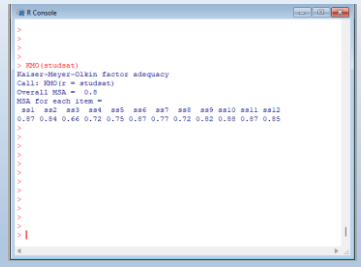
Factor Analysis



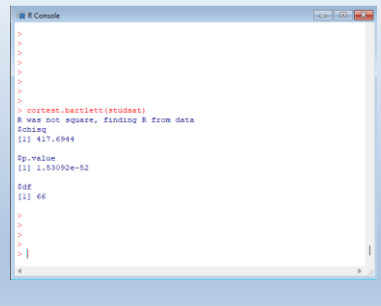
78

Factor Analysis



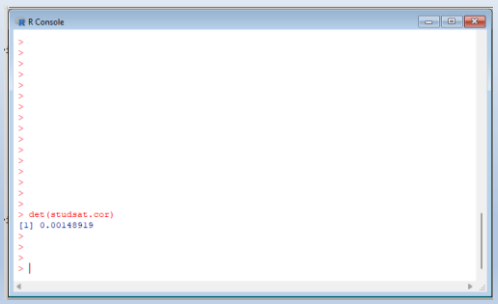
79

Factor Analysis



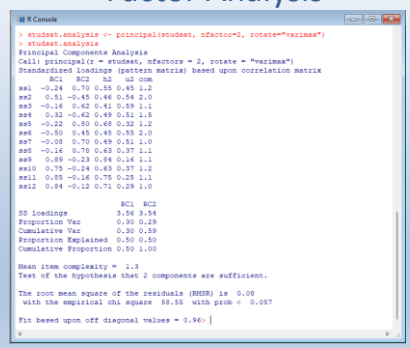
80

Factor Analysis



81

Factor Analysis



82

Factor Analysis

	RC1	RC2	h2	u2	com
ss1	-0.24	0.70	0.55	0.45	1.2
ss2	0.51	-0.45	0.46	0.54	2.0
ss3	-0.16	0.62	0.41	0.59	1.1
ss4	0.32	-0.62	0.49	0.51	1.5
ss5	-0.22	0.80	0.68	0.32	1.2
ss6	-0.50	0.45	0.45	0.55	2.0
ss7	-0.08	0.70	0.49	0.51	1.0
ss8	-0.16	0.78	0.63	0.37	1.1
ss9	0.89	-0.23	0.84	0.16	1.1
ss10	0.75	-0.24	0.63	0.37	1.2
ss11	0.85	-0.16	0.75	0.25	1.1
ss12	0.84	-0.12	0.71	0.29	1.0

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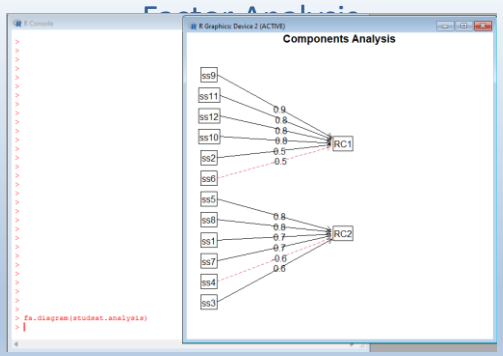
Factor Analysis

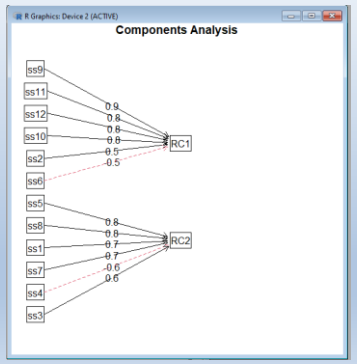
	RC1	RC2
SS loadings	3.56	3.54
Proportion Var	0.30	0.29
Cumulative Var	0.30	0.59
Proportion Explained	0.50	0.50
Cumulative Proportion	0.50	1.00

84



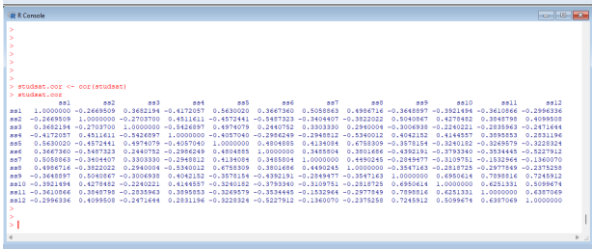
85

Factor Analysis



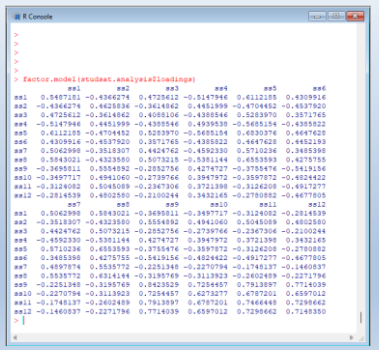
86

Factor Analysis



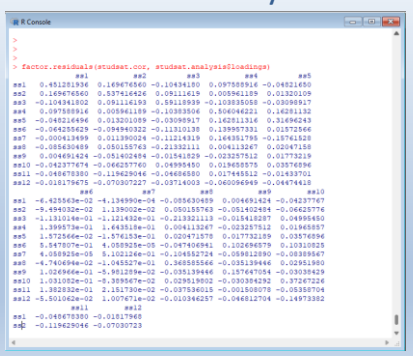
87

Factor Analysis



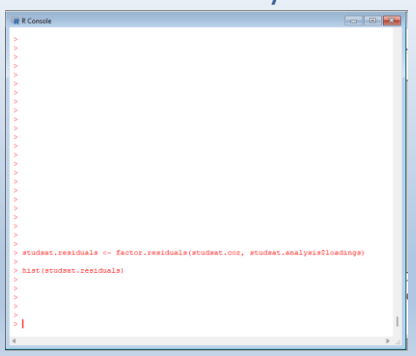
88

Factor Analysis



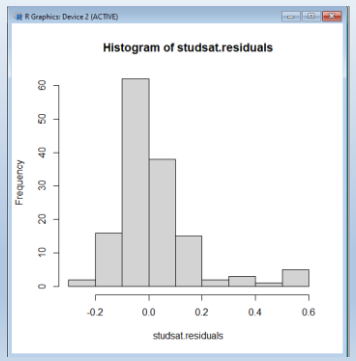
89

Factor Analysis



90

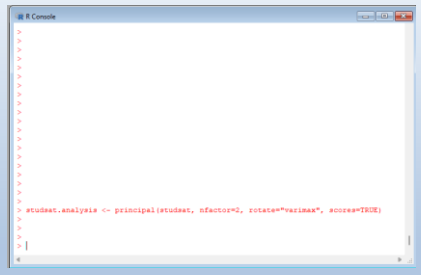
Factor Analysis



91

91

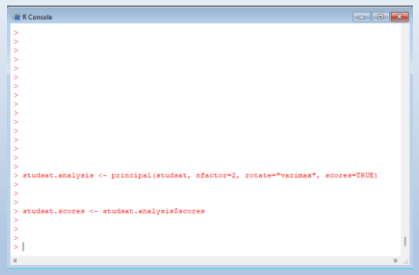
Factor Analysis



92

92

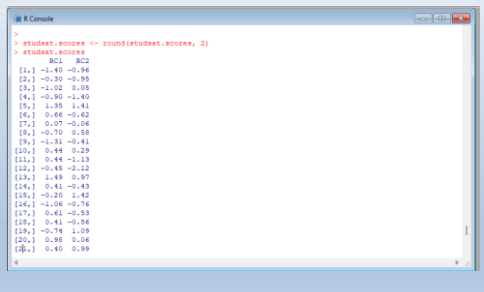
Factor Analysis



93

93

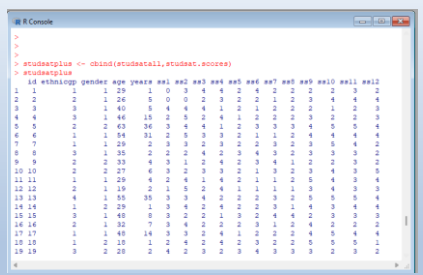
Factor Analysis



94

94

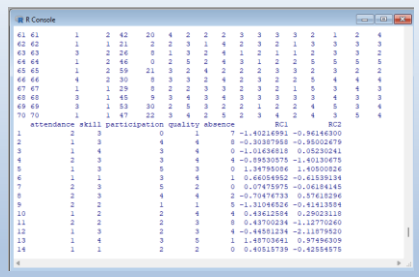
Factor Analysis



95

95

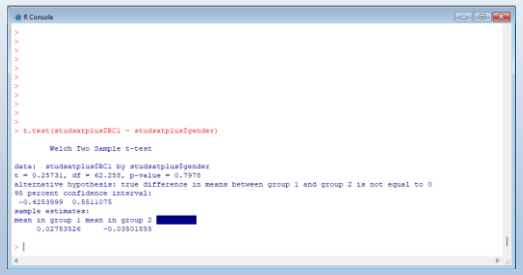
Factor Analysis



96

96

Factor Analysis



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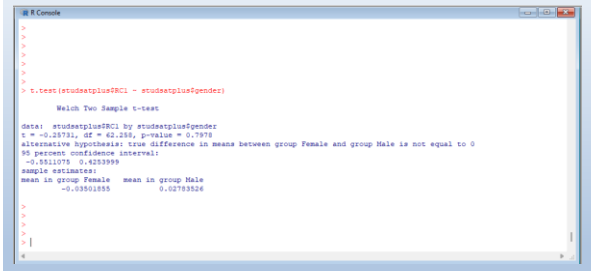
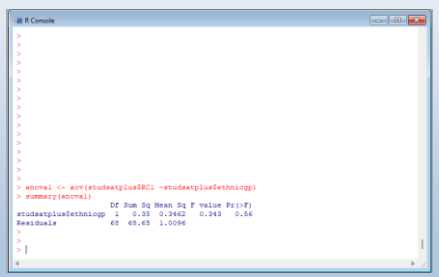
Factor Analysis



98

Factor Analysis



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Factor Analysis

The 12 items of the Job Satisfaction Scale were subjected to factor analysis with PCA and varimax rotation using R version 4.3.2. Prior to performing the factor analysis, the suitability of data for factor analysis was assessed. Inspection of the correlation matrix revealed the presence of several coefficients of .3 and above. The Kaiser-Meyer-Olkin value was .80, exceeding the recommended value of .6 (Kaiser 1960) and Bartlett's Test of Sphericity (Bartlett 1954) reached statistical significance, supporting the factorability of the correlation matrix.

Factor analysis revealed the presence of two components with eigenvalues exceeding 1, explaining 30% & 29% of the variance respectively. An inspection of the screeplot revealed a clear after the second component. Using Catell's (1966) scree test, it was decided to retain two components for further investigation.

The two solution explained a total of 59% of the variance, with Component 1 contributing 30% and Component 2 adding another 29% . To aid in the interpretation of these three components, varimax rotation was performed. The rotated solution revealed the presence of simple structure, with all components showing a number of strong loadings and most variables loading substantially on only one component.

A test was conducted to see whether the factor model is sufficient to explain the observed data. The null hypothesis is that a 2-factor model is sufficient. For this model, the chi-square statistic is 58.55. The p-value for the chi-square test is 0.057 which is larger than .05. Therefore, we fail to reject the null hypothesis that the factor model have a good fit to the data.

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References & Further Reading

Bartlett, M. S. (1954). A note on the multiplying factors for various chi square approximations. *Journal of the Royal Statistical Society, 16* (Series B), 296-8.

Cattell, R. B. (1966). The scree test for the number of factors. *Multivariate Behavioral Research, 1,* 245-276.

Colman, A. M. & Pulford, B. D. (2008). *A crash course in SPSS for Windows* (4th ed.). Oxford: Blackwell

Dörnyei, Z. (2007). *Research methods in applied linguistics.* Oxford: Oxford University Press.

De Vries, A. & Meys, J. (2015). *R for dummies* (2nd ed.). London: Wiley.

Field, A., Miles, J. & Field, Z. (2013). *Discovering statistics using R.* London: Sage.

Hatch, E. & Lazaraton, A. (1991). *The research manual: Design and statistics for applied linguistics*. Boston, MA: Heinle & Heinle.

Hayton, J. C., Allen, D. G. & Scarpello, V. (2004). Factor retention decisions in exploratory factor analysis: A tutorial on parallel analysis. *Organizational Research Methods 7,* 191-204.

Kabacoff, R. I. (2015). *R in action* (2nd ed.). Shelter Island, NY: Manning.

Kaiser, H. F. (1960). The application of electronic computers to factor analysis. *Educational and Psychological Measurement, 20*, 141-151.

Larson-Hall, J. (2016). *A guide to doing statistics in second language research using SPSS and R* (2nd ed.). London: Routledge.

Loewen, S. & Gonulal, T. (2015). Exploratory factor analysis and principal component analysis. In L. Plonsky (Ed.), *Advancing quantitative methods in second language research* (pp. 182-212). London: Routledge.

Loewen, S., Lavolette, E., Spino, L., Papi, M., Schmidtke, J., Sterling, S., & Wolff, D. (2014). Statistical literacy among applied linguists and second language acquisition researchers. *TESOL Quarterly, 48,* 360-388.

Lowie, W. & Seton, B. (2013). *Essential statistics for applied linguistics*. London: Palgrave Macmillan.

Popham, W. J. & Sirotnik, K. A. (1973). *Educational statistics: Use and interpretation* (2nd ed.). New York: Harper & Row

Tabachnick, B. G. & Fidell, L. S. (2007). *Using multivariate statistics.* (5th ed.). Boston: Allyn & Bacon.

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